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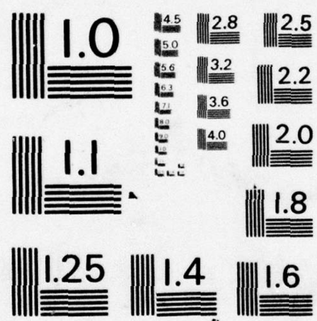

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September 1976

MODAL REPRESENTATION OF A HORIZONTAL WIRE ABOVE A FINITELY CONDUCTING EARTH

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20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

The modal representation of the fields of a thin horizontal wire (with or without dielectric coating) above the surface of a finitely conducting earth is discussed. A general expression for the excitation of the discrete modes by an aperture field of arbitrary form in the transverse plane is given. As a consequence, it is proved that any discrete mode can be excited with high efficiency with an aperture of finite size. The dependence of the discrete modal propagation constants of the discrete modes upon various physical parameters is investigated in some detail and the possibility of mode degeneration is demonstrated. Field plots of the

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cont.

discrete modes are presented which show the qualitative differences between the so-called "transmission-line" mode and the recently discovered "earth-attached" mode.

I. Introduction

The problem of wave propagation along a horizontal wire parallel to the earth's surface was first investigated nearly half a century ago by Carson, Pollaczek, and Haberland [1-5] in a low frequency approximation suitable for application to overhead power lines. Since that time, various refinements to the low frequency theory have been made [6-8] of which excellent summaries can be found in the books [9,10]. However, it was eventually desired to determine higher frequency characteristics of such wires, in order to accurately account for the earth's influence on elevated antennas, as well as to describe transient processes on overhead power lines. Thus, about twenty years ago, more rigorous analyses of the problem began to appear [10-25]. (See also the additional bibliography at the end of the reference list.)

With few exceptions [10, 11, 14, 21-24], the excitation of the wire has not been considered, and even these are limited to a delta-function voltage generator [10, 14, 22-24], plane-wave incidence [21], or a purely formal treatment of a magnetic ring current [11]. The remainder of the work has sought to find modal solutions for the source-free case, by postulating a current distribution $I \exp\{i\omega t - \Gamma z\}$, where Γ is an unknown propagation constant along the wire, evaluating the resultant primary fields and secondary (induced by the earth) fields, and enforcing the boundary conditions at the wire to obtain an eigenvalue equation for Γ. In its most rigorous formulation [10-17, 20], this equation is an implicit transcendental equation accurately solvable only by numerical methods. Only under suitable approximations [12-15, 17-20] can a closed form expression for Γ be obtained.

The modal approach in essence is an interpretation of the electromagnetic characteristics of the wire from a waveguide point of view. This structure, being both open and lossy, can possess a quite complicated set of modal properties [26]. We may expect the discrete modes to be finite in number, and thus require an additional set of continuous or radiation modes to form a complete modal set in terms of which an arbitrary field of a source may be expanded. Indeed, we may find that improper leaky modes may exist on this structure, as has been claimed by some researchers [16,25]. Alternatively, it is possible that the mode set of this structure does not form a complete set. This is a well-known occurrence in the theory of nonselfadjoint operators [27]. In fact, one could call into question the physical reality of the modes themselves on the basis of the fact that, if $\Gamma = \alpha + i\beta$, with $\alpha > 0$, the assumed current distribution on the wire must become infinite at $z = -\infty$.[1]

In view of such difficulties, the most rigorous approach to the problem would seem to be the analysis of a wire of finite length and with finite excitation by an integral equation approach typical of antenna theory. That this approach possesses its own difficulties is shown by the work of [10,23] in which a number of approximations is made until the current distribution along the wire becomes essentially that of a standing wave with a wave number identical to that of the discrete mode found by the aforementioned infinite wire analyses [15,17]. Apparently, then, the modal viewpoint is not unrelated to the antenna theory viewpoint for an

[1] Even on a lossless structure for which $\alpha = 0$, one could argue that the assumed current distribution violates the outgoing radiation condition at $z = -\infty$, unless some source were postulated to exist there.

antenna of reasonable length. On the other hand, one may argue from the analysis of an imperfectly conducting coaxial line, for example, that, given an arbitrarily long line, the current on the wire would eventually be dominated by radiation currents instead of "modal" currents which usually decay in an exponential fashion [28]. Thus, a discrete modal representation is useful only if the length of the wire is such that the current along the entire length of the wire can be adequately described by discrete "modal currents" for a prescribed excitation of finite extent.

The foregoing discussion points up the need for a rigorous, general analysis of the modal representation of the fields on infinite thin-wire structures parallel to the air-earth interface. Thus, we seek a general expression for the excitation of the modes of propagation along the wire by an aperture field of arbitrary form in the transverse plane. Some further numerical investigation of the behavior of the discrete modal propagation constants is given in Section III. We also present some field plots for the discrete modes for a variety of wire parameters are presented, and the differences between the modes are examined in some detail. A knowledge of the modal pecularities is vital to the ability to excite one of them with any efficiency.

II. Excitation of a horizontal wire above the ground by a given current distribution

Let us consider the problem of exciting an infinite thin wire parallel to and above the earth by a prescribed set of electric and magnetic current sources $\bar{J}_e^{ext}$ and $\bar{J}_m^{ext}$ in the region outside the wire as shown in Fig. 1(a). The height of the center of the wire above the earth is h, the radius of the wire is a, the region above the earth

$(z > 0)$ is taken to be free space with permittivity ε_o and permeability μ_o, while the earth $(z < 0)$ is considered to have a complex dielectric constant $\varepsilon = n^2\varepsilon_o$, where n is a complex refractive index. A time dependence of $\exp(-i\omega t)$ will be assumed.

The solution of this problem can be applied to some other related problems as well. If the wire is semi-infinite $(z > 0)$ as shown in Fig. 1(b), the effect of the sources can be represented approximately in terms of the aperture field $\bar{E}_t^o$ produced by the sources at $z = 0$. This would be an idealization of the physical situation sketched in Fig. 1(c), where the wire is driven by a voltage generator of some kind. Following Harrington [29], we may replace the sources and aperture field by a perfect (electric) conductor at $z = 0$ with an adjacent magnetic current sheet $\bar{E}_t^o \times \bar{a}_z$ located at $z = 0^+$. This in turn is equivalent to removing the conductor and imaging the magnetic current sheet (as well as the wire and the medium) to give a total current $\bar{J}_m^o\delta(z) = 2\bar{E}_t^o \times \bar{a}_z\delta(z)$ in the aperture plane exciting an infinite wire over the earth (Fig. 1(d)).

The wire will be assumed to be thin compared to a wavelength and to the height above ground:

$$A = k_1 a \ll 1; \quad A \ll H = k_1 h$$

where $k_1 = \omega\sqrt{\mu_o\varepsilon_o}$ is the wavenumber of free space. Under these conditions, the equivalent total current $I(z)$ may be taken to be the average over the circumference of the wire of the H field component multiplied by $2\pi a$ [14]. The average value of the E_z field component at the circumference of the wire will then consist of three parts. The first contribution is from the induced (unknown) equivalent current $I(z)$ [11,14,15,24]:

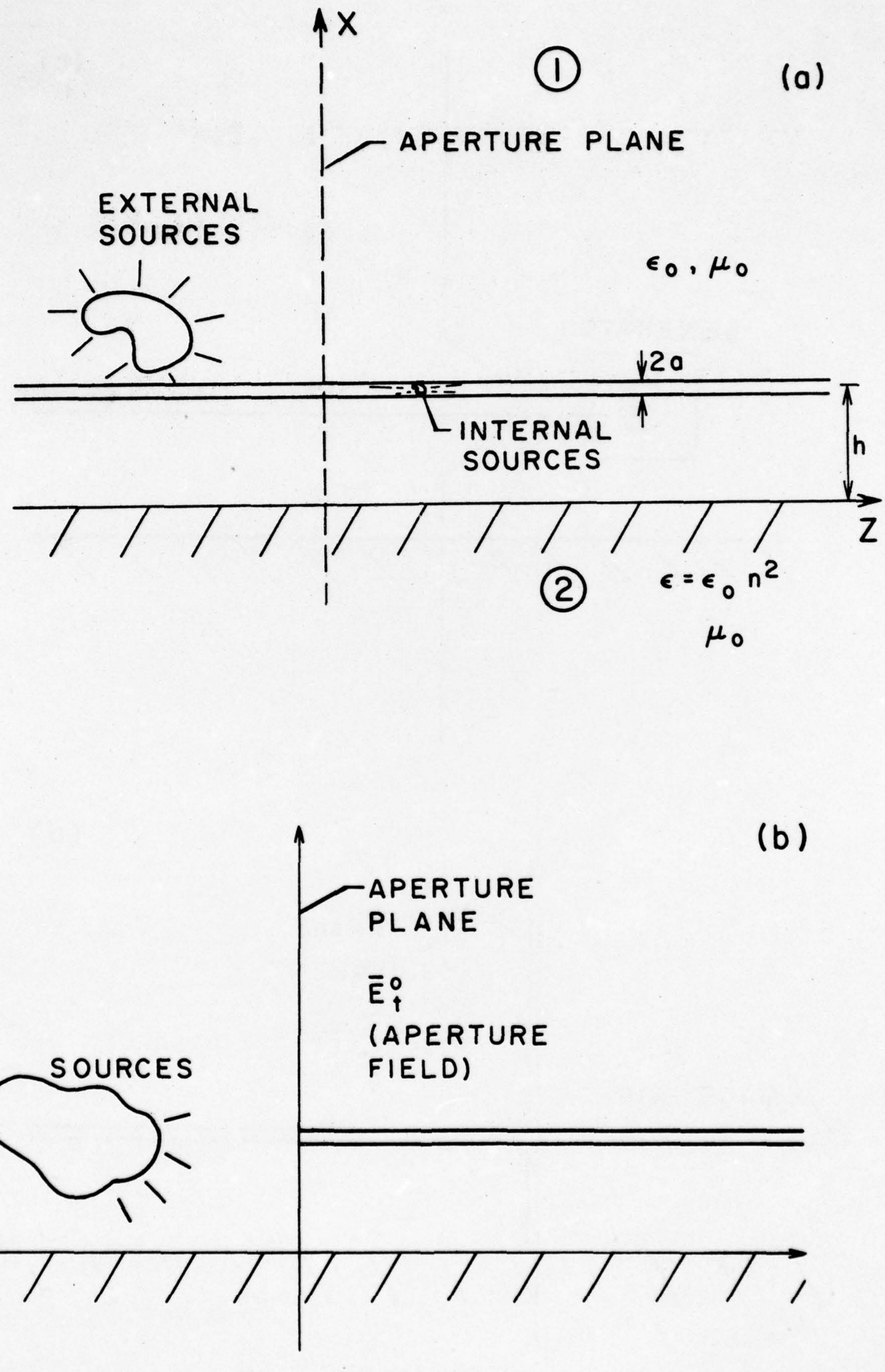
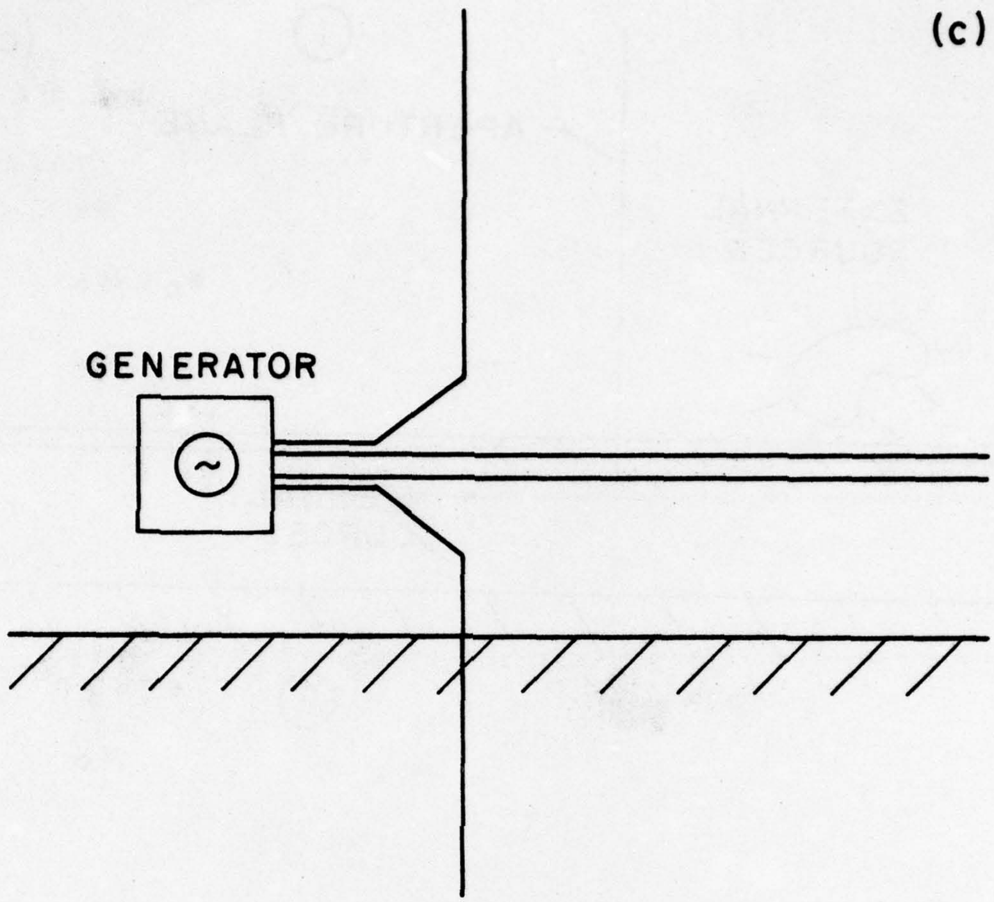

X
①
(a)
APERTURE PLANE
EXTERNAL SOURCES
ϵ_0, μ_0
2a
INTERNAL SOURCES
h
Z
②
$\epsilon = \epsilon_0 n^2$
μ_0
(b)
APERTURE PLANE
$\bar{E}^o_t$
(APERTURE FIELD)
SOURCES

Figure 1a,b


(c)
GENERATOR

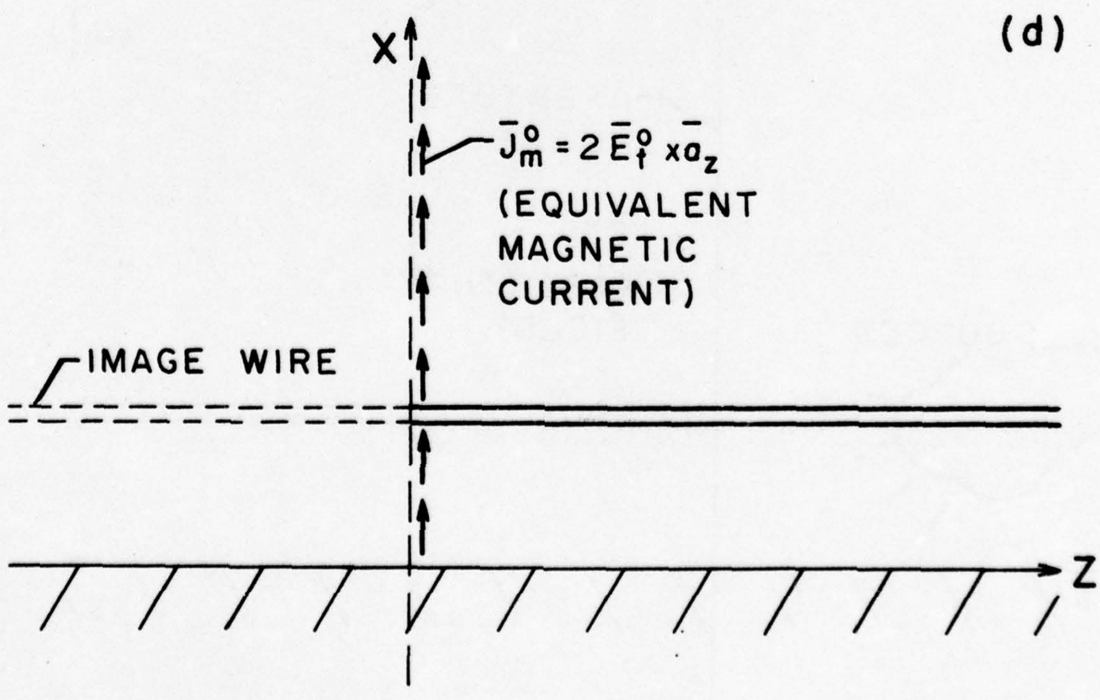

(d)
X
$\bar{J}_m^o = 2\bar{E}_t^o \times \bar{a}_z$
(EQUIVALENT
MAGNETIC
CURRENT)
IMAGE WIRE
Z

Figure 1c,d

$$-\frac{\eta_o k_1^2}{8\pi} \int_{-\infty}^{\infty} I(z')M_o(z-z')dz' \tag{1}$$

where $\eta_o = \sqrt{\mu_o/\varepsilon_o}$ is the wave impedance of free space. The kernel $M_o(z-z')$ can be given in terms of its Fourier transform $\tilde{M}(\alpha)$:

$$M_o(z-z') = \int_{-\infty}^{\infty} \tilde{M}_o(\alpha)\exp[ik_1\alpha(z-z')]d\alpha \tag{2}$$

where

$$\tilde{M}_o(\alpha) = J_o(A\zeta)\{\zeta^2[H_o^{(1)}(A\zeta) - H_o^{(1)}(2H\zeta)J_o(A\zeta)] + J_o(A\zeta)[P(\alpha)-Q(\alpha)]\} \tag{3}$$

For thin wires, it is usually permissible to take $J_o(A\zeta) \simeq 1$, however, if it is desired to include sources internal to the wire (such as a delta-function generator), the product $H_o^{(1)}(A\zeta)J_o(A\zeta)$ must be left intact in order to correctly describe the singularity at the source [14]. The first square brackets in (3) are the contributions from the wire and its perfect image; the functions $P(\alpha)$ and $Q(\alpha)$ are corrections due to the finite conductivity of the earth:

$$P(\alpha) = \frac{2}{i\pi} \int_{-\infty}^{\infty} \frac{\exp(-2Hu_1)}{u_1+u_2} d\lambda \tag{4}$$

$$Q(\alpha) = \frac{2\alpha^2}{i\pi} \int_{-\infty}^{\infty} \frac{\exp(-2Hu_1)}{u_2+n^2u_1} d\lambda \tag{5}$$

In (3)-(5), we have defined

$$\begin{aligned} u_1 &= (\lambda^2 - \zeta^2)^{\frac{1}{2}} & \quad \mathrm{Re}(u_1) \geq 0 \\ u_2 &= (\lambda^2 - \zeta_n^2)^{\frac{1}{2}} & \quad \mathrm{Re}(u_2) \geq 0 \end{aligned} \tag{6}$$

$$\zeta = (1-\alpha^2)^{\frac{1}{2}} \quad \mathrm{Im}(\zeta) \geq 0; \qquad \zeta_n = (n^2-\alpha^2)^{\frac{1}{2}} \quad \mathrm{Im}(\zeta_n) \geq 0$$

For the integral (2), we choose the path so that

$$\zeta = (1-\alpha^2)^{\frac{1}{2}} \qquad |\alpha| < 1$$

$$= i(\alpha^2-1)^{\frac{1}{2}} \qquad |\alpha| > 1$$

This is the original contour shown in Fig. 2.

A second contribution to the average E_z at the wire is due to the external sources $\bar{J}_e^{ext}$ and $\bar{J}_m^{ext}$ (still in the absence of the wire-the boundary condition at the wire surface has not yet been enforced). Using the reciprocity principle [29], this "incident" E_z (the first contribution may be thought of as "scattered") can be expressed in terms of the fields due to a source at the center of the wire:

$$\iiint_V [\bar{E}_o^w(\bar{x}'';\bar{x})\cdot\bar{J}_e^{ext}(\bar{x}'') - \bar{H}_o^w(\bar{x}'';\bar{x})\cdot\bar{J}_m^{ext}(\bar{x}')]dV \tag{7}$$

Here V is the volume containing the external sources and $\bar{E}_o^w(\bar{x}'';\bar{x})$ and $\bar{H}_o^w(\bar{x}'';\bar{x})$ are the fields produced at the point $\bar{x}''$ due to an equivalent (averaged) z-directed electric current source $\bar{J}_e = \bar{a}_z\delta(x''-h)\delta(y'')D(z''-z)$ at the center of the wire $\bar{x} = (h,0,z)$, where $D(z)$ is given in Appendix A and represents the effect of averaging a unit z-directed current source on the wire surface around the circumference of the wire [14] (so that $D(z) \to \delta(z)$ as $A \to 0$). $\bar{E}_o^w$ and $\bar{H}_o^w$ are given for reference in Appendix A. Equation (6) allows us to formulate the solution for arbitrary excitation solely in terms of the fields of z-directed electric currents.

We shall suppose that the boundary conditions at the surface of the wire can be put in the form

$$\langle E_z\rangle = \frac{k_1}{2\pi}\int_{-\infty}^{\infty} I(z')Z_s(z-z')dz' - V^{eq}(z) \tag{8}$$

The term V^{eq} allows for the possibility of sources internal to the wire, $\langle E_z \rangle$ denotes the field averaged around the circumference of the wire, and the use of the surface impedance convolution operator $Z_s(z-z')$ assumes that the fields at the wire surface are essentially azimuthally independent and TM [30]. Hence, this boundary condition should be expected to be valid in many thin-wire applications (see the discussion in Section III). In this manner, one can characterize a wire of finite conductivity [10,12,13,19,31] as well as a thin dielectric coated wire (Goubau line) [10,19,20,32,33] or a cable with a leaky coaxial braid [30,34].

Combining (1) and (6) and enforcing boundary condition (8) leads to the following integral euqation for $I(z)$:

$$\int_{-\infty}^{\infty} I(z')M(z-z')dz' = \frac{8\pi}{\eta_o k_1^2}\left\{\iiint_V [\bar{E}_o^w(\bar{x}'';\bar{x})\cdot\bar{J}_e^{ext}(\bar{x}'') - \bar{H}_o^w(\bar{x}'';\bar{x})\cdot\bar{J}_m^{ext}(\bar{x}'')]dV'' + V^{eq}(z)\right\} \tag{9}$$

where $\bar{x} = (h,0,z)$ and

$$M(z-z') = M_o(z-z') + \frac{4}{\eta_o k_1} Z_s(z-z') \tag{10}$$

This equation can be solved by applying the convolution theorem

$$\frac{k_1}{2\pi}\int_{-\infty}^{\infty} f(z')g(z-z')dz' = \int_{-\infty}^{\infty} \tilde{f}(\alpha)\tilde{g}(\alpha)\exp(ik_1\alpha z)d\alpha \tag{11}$$

to equation (9) and taking its Fourier transform with respect to z, resulting in

$$\tilde{I}(\alpha) = \frac{4}{\eta_o k_1 \tilde{M}(\alpha)}\left\{\iiint_V [\tilde{\bar{E}}_o^w(\bar{x}_t'';-\alpha)\cdot\bar{J}_e^{ext}(\bar{x}'') - \tilde{\bar{H}}_o^w(\bar{x}_t'';-\alpha)\cdot\bar{J}_m^{ext}(\bar{x}'')]e^{-ik_1\alpha z''}dV'' + \tilde{V}^{eq}(\alpha)\right\} \tag{12}$$

where $\bar{x}_t''$ is the transverse (to z) part of $\bar{x}'' = (\bar{x}_t'',z'')$ and the Fourier transforms are given by

$$\tilde{I}(\alpha) = \frac{k_1}{2\pi} \int_{-\infty}^{\infty} I(z)\exp(-ik_1\alpha z)dz \tag{13}$$

and similarly for $\tilde{M}(\alpha), \tilde{Z}_s(\alpha), \tilde{V}^{eq}(\alpha), \tilde{\bar{E}}_o^w(\bar{x}_t'';\alpha)$ and $\tilde{\bar{H}}_o^w(\bar{x}_t'';\alpha)$ (the latter two may be found explicitly in Appendix A). The term $\tilde{Z}_s(\alpha)$, defined as the Fourier transform of $\tilde{Z}_s(z)$, is the axial impedance of the wire.

In the special case when $\bar{J}_e^{ext} \equiv \bar{J}_m^{ext} \equiv 0$ and $V^{eq} = V\delta(z)$, $\tilde{V}^{eq}(\alpha) = k_1/2\pi$ and we recover

$$I(z) = \int_{-\infty}^{\infty} \tilde{I}(\alpha)\exp(ik_1\alpha z)d\alpha = \frac{2}{\pi\eta_o} \int_{-\infty}^{\infty} \frac{\exp(ik_1\alpha z)d\alpha}{\tilde{M}(\alpha)} \tag{14}$$

as found in [24]. Likewise if $\bar{J}_e^{ext}$ or $\bar{J}_m^{ext}$ is set equal to a constant vector times $\delta(\bar{x}''-\bar{x}_o)$, where $\bar{x}_o$ is some point in space, we recover the solution for the arbitrarily oriented electric or magnetic Hertzian dipole source located at $\bar{x}_o$. The expression for a VED agrees with the result of more direct derivations [35,36].

Of particular interest to us is the special case mentioned above in connection with Fig. 1(d). Here

$$\bar{J}_m^{ext} = 2\bar{E}_t^o \times \bar{a}_z\delta(z'') \; ; \quad \bar{J}_e^{ext} = 0 \; ; \quad V^{eq} = 0 \tag{15}$$

corresponds to an aperture field distribution $\bar{E}_t^o$ in the plane $z = 0$. For this case (12) becomes

$$\tilde{I}(\alpha) = \frac{8}{\eta_o k_1 \tilde{M}(\alpha)} \iint_S \bar{a}_z \cdot [\bar{E}_t^o(\bar{x}_t'') \times \tilde{\bar{H}}_o^w(x''_t;-\alpha)]dS \tag{16}$$

where S is the (infinite) aperture plane $z = 0$. Inserting (16) into the

inverse Fourier transformation and assuming the order of integration can be interchanged, we obtain

$$\tilde{I}(z) = \frac{8}{\eta_o k_1} \iint_S \bar{a}_z \cdot \left\{ \bar{E}_t^o(\bar{x}_t'') \times \int_{-\infty}^{\infty} \frac{\tilde{\bar{H}}_o^W(\bar{x}_t''; -\alpha) \exp(ik_1 \alpha z)}{\tilde{M}(\alpha)} d\alpha \right\} dS'' \tag{17}$$

Following [24], we extend the definitions of functions of α into the complex α-plane as indicated following (5), specifying also $0 < \arg W \leq \pi$, where $W = (\alpha_B^2 - \alpha^2)^{\frac{1}{2}}$, $\alpha_B = n/(n^2+1)^{\frac{1}{2}}$ because of the presence of this third branch cut in the α-plane [17]. For $z > 0$, we deform the contour of integration into the upper half of the α-plane. The singularity structure of the integrand is determined by $\tilde{M}(\alpha)$ ($\bar{H}_o^W$ has only branch cuts already possessed by $\tilde{M}(\alpha)$), which consists of three branch cuts[2] and a number of poles[3] in each of the upper and lower half-planes. The contour deformation splits (17) into four contributions (if the branch cut at $\alpha = n$ is neglected):

$$I(z) = I_{p1}(z) + I_{p2}(z) + I_{B1}(z) + I_{B2}(z) \tag{18}$$

where

$$I_{p1,2}(z) = \frac{16\pi i}{\eta_o k_1} \frac{\exp(ik_1 \alpha_{p1,2} z)}{\tilde{M}'(\alpha_{p1,2})} \int_S \bar{a}_z \cdot [\bar{E}_t^o(\bar{x}_t'') \times \tilde{\bar{H}}_o^W(\bar{x}_t''; -\alpha_{p1,2})] dS'' \tag{19}$$

$$I_{B1}(z) = -\frac{8}{\eta_o k_1} \int_S \bar{a}_z \cdot \left\{ \bar{E}_t^o \times \int_0^{\infty} \left[\frac{\bar{H}_o^{W1}(\bar{x}_t''; -\alpha)}{\tilde{M}^1(\alpha)} - \frac{\bar{H}_{o\pi}^{W1}(x_t''; -\alpha)}{M_\pi^1(\alpha)} \right] \exp(ik_1 \alpha z) d\zeta \right\} dS'' \tag{20}$$

---

[2] The contribution from the branch cut associated with the square-root of $(\alpha^2 - n^2)$ is usually neglected under the approximation of a large refractive index of earth, i.e. $|n|^2 >> 1$, although one need not make such an approximation.

[3] Usually only two in previous investigations [17]. It is seen in section III that the two poles can degenerate into one double pole. This special case is handled in Appendix B.

$$I_{B2}(z) = -\frac{8}{\eta_o k_1}\int_S \bar{a}_z\cdot\left\{\bar{E}_t^o\times\int_0^\infty\left[\frac{\tilde{\bar{H}}_o^{w2}(\bar{x}_t'';-\alpha)}{\tilde{M}^2(\alpha)} - \frac{\tilde{\bar{H}}_o^{w2}(x_t'';-\alpha)}{\tilde{M}_\pi^2(\alpha)}\right]\exp(ik_1\alpha z)dW\right\} dS'' \tag{21}$$

Here $\alpha_{p1,2}$ are the surface wave poles, the superscript 1 in (20) indicates branch cut and the subscript π denotes $\arg\zeta = \pi$. Similarly the superscript 2 in (21) indicates branch cut 2 and the subscript π denotes $\arg W = \pi$. The situation in the complex α-plane is shown in Fig. 2.

From the definition of $\bar{H}_o^w$ it is clear that this decomposition is into contributions from two discrete modal currents and two continuous spectra of radiation currents (the branch cut integrals.) Although the fields of continuous spectrum modes do not satisfy the radiation condition individually, they are individually bounded at infinity, and when excited by a finite source, they must collectively satisfy the radiation condition. It is shown in Appendix C that if $\bar{E}_t^o$ is replaced by the modal field corresponding to a discrete mode, then the only nonvanishing contribution to (18) is the corresponding discrete modal current.

Now suppose $\bar{E}_t^o$ were arbitrarily close to the E-field of one of the discrete modes (which decays at infinity in the transverse plane). For definiteness, let us say that $\bar{E}_t^o$ is the field pattern of the discrete mode of interest, truncated at a circle of finite radius R, and equal to zero for $\rho = |\bar{x}_t| > R$ in the transverse aperture plane ($z = 0$). $\bar{E}_t^o$ then differs from the exact modal field by a quantity (see footnote 3) of order $\exp(ik_1\zeta_p R)$ or $\exp(ik_1 W_p R)$ (whichever is larger), where $\zeta_p = (1-\alpha_p)^{\frac{1}{2}}$ and $W_p = (\alpha_B^2 - \alpha_p^2)^{\frac{1}{2}}$ define a transverse wavenumber and a "spreading" wavenumber (describing the behavior of the mode as $y \to \pm\infty$ along the air-earth interface)

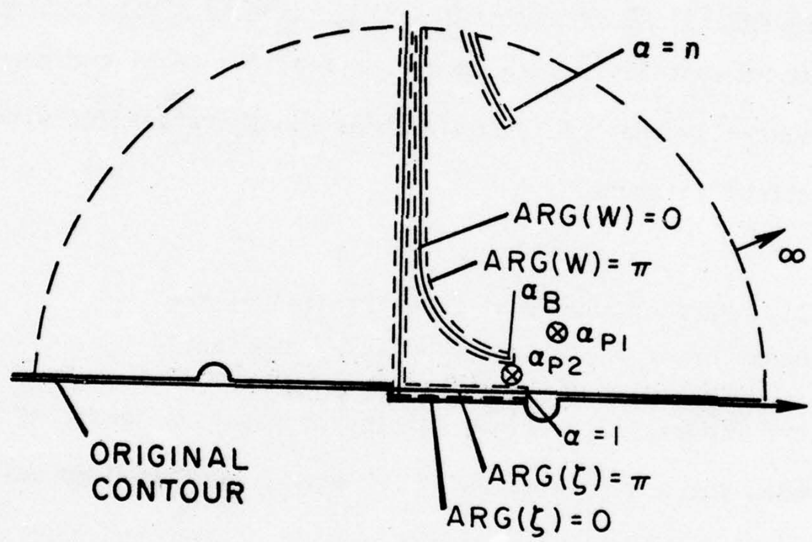


Figure 2

Contour deformation in the α-plane.

$W = (\alpha_B^2 - \alpha^2)^{\frac{1}{2}}$; $\mathrm{Im}(W) > 0$; $\zeta = (1-\alpha^2)^{\frac{1}{2}}$; $\mathrm{Im}(\alpha) > 0$.

characteristic of the given mode. Using orthogonality, it is then possible to bound the quantities (20) and (21) by a value which tends exponentially to zero (recalling the choice of Riemann sheets for W and ζ) as $R \to \infty$. Hence using only orthogonality, without assuming completeness of the modes, it has been demonstrated that either of the discrete modes can in principle, be excited with arbitrarily high efficiency by taking a large enough truncated replica of the modal E-field as the aperture excitation. It is then apparent that given such an excitation, the total current on the wire is essentially that of the mode under consideration for wire lengths of practical interest.

III. Further numerical investigation of the behavior of the modal propagation constants

In a previous report [17], values for the propagation constants of the two discrete modes for a single value of n and a and various values of h were presented for the case of a bare wire. These had been calculated numerically by using approximations of the Sommerfeld integrals appearing in the modal equation in terms of known analytic functions. The approximations were based on the assumption of a highly but finitely conducting earth and sufficiently large height H. In order to verify the accuracy of the approximate modal equation, we compared the solutions of the modal equation obtained from the approximate one, to those obtained from the exact one, wherein the Sommerfeld integrals were evaluated by numerical integration. More specifically, the "exact" modal equation used is of the form $\tilde{M}_o(\alpha) = 0$, where $\tilde{M}_o$ is given by (3). An iterative scheme similar to the commonly used Newton's method algorithm was used to find

the roots of the exact equation. The results for a single bare wire, to seven digit accuracy, are shown in Table 1. It is particularly gratifying to see that the results obtained from the approximate modal equation are excellent for the largest h (within 10^{-5}, compared to those obtained from the exact equation) and even for the smallest h, still gives errors of less than 10^{-3}. Within the limits of validity outlined by these results, it is felt this comparison provides adequate justification for use of either exact or approximate modal equations in subsequent calculations.

It seems appropriate at this point to discuss the only approximation used in deriving the modal equation, i.e., that the current on the wire may be taken to be z-directed and averaged over the wire surface, and that an average boundary condition around the circumference may be used. This would hold exactly for a symmetrically excited wire in the absence of the earth, but due to its influence, the so-called "proximity effect" will cause the wire current to have higher-order Fourier components in its angular distribution around the wire, as well as azimuthally directed components.

For a two-wire transmission line with imperfect conductors the effect of higher-order Fourier components was taken into account in a classic paper of Mie [37] (a more accessible source is Sommerfeld [38]; for a generalization to arbitrary noncircular conductors, see [39]). For the bare, imperfectly conducting wire over the earth, some discussion of the relative error in E_z incurred by the neglect of the first higher terms of the Fourier series is done in [12], but a more complete treatment has recently been done by Pogorzelski and Chang [40]. For the parameter values used here, these errors can typically be shown to be much less than one percent.

TABLE I

$n = 5.3 + i0.45$ $a = .007\lambda$

	α_{approx}	α_{exact}	h/λ
α_{p1}	$.99554672 + i3.499775 \times 10^{-2}$	$.99522062 + i3.456630 \times 10^{-2}$	.125
α_{p2}	$.98795820 + i2.210481 \times 10^{-3}$	$.98806262 + i2.190248 \times 10^{-3}$	.125
α_{p1}	$.99052338 + i1.564676 \times 10^{-2}$	$.99046228 + i1.562346 \times 10^{-2}$	.24
α_{p2}	$.99240980 + i2.414344 \times 10^{-3}$	$.99245040 + i2.392416 \times 10^{-3}$	.24
α_{p1}	$.98862947 + i1.002956 \times 10^{-2}$	$.98861684 + i1.003723 \times 10^{-2}$	.35
α_{p2}	$.99555888 + i2.047218 \times 10^{-3}$	$.99556699 + i2.028685 \times 10^{-3}$	.35
α_{p1}	$.98791384 + i6.994975 \times 10^{-3}$	$.98791378 + i6.999463 \times 10^{-3}$	.5
α_{p2}	$.99753534 + i1.157296 \times 10^{-3}$	$.99753509 + i1.149982 \times 10^{-3}$	.5

Comparison of the approximate and the exact, normalized (complex) propagation constants of the two discrete modes, α_{p1} and α_{p2}, for a single, conducting wire above earth, a = radius of wire, h = wire height, λ = free space wavelength, n = refractive index of earth.

This discussion emphasizes that a previous particular result obtained by Coleman for the limiting case of $h \to 0$ [8,15,20] must be understood as a legitimate limiting value only when the ratio $a/2h$ is maintained smaller than, say, 0.1, as the limit $h \to 0$ is taken. It is unclear what effect a small but finite wire radius might have upon the propagation constant of a wire located in the interface.

In Fig. 3 the solutions of the bare wire modal equation $\tilde{M}_o(\alpha) = 0$ are plotted for several values of a and h, with $n = 5.3 + i0.95$ (the loss tangent for this case is about twice that for the case of Table I). Between the values $a = 10^{-3}\lambda$ and $a = 1.66 \times 10^{-4}\lambda$, an interesting phenomenon is seen to have occurred. At some value h in the neighborhood of 0.2λ, there must be a pair of curves for some value of a between those of curves 2 and 3 which actually have this value of h in common. This phenomenon is <u>modal degeneration</u> *[41-43] which* is known to appear in the earth-ionosphere waveguide [44] and in the earth-crust waveguide [45], for example. Further discussion will be reserved for the conclusion.

Figure 4 presents data for a dielectric-coated wire (Goubau line) located above the earth. Here the modal equation is $\tilde{M}(\alpha) = 0$, with the $Z_s(\alpha)$ corresponding to (10) given by [10, 19, 20, 32, 33]:

$$\tilde{Z}_s(\alpha) = -\frac{k_1\eta_o}{4}\,\frac{\zeta\zeta_r}{n_r^2}\,H_1^{(1)}(\zeta B)J_o(\zeta B)\,\frac{J_o(\zeta_r B)H_o^{(1)}(\zeta_r A) - J_o(\zeta_r A)H_o^{(1)}(\zeta_r B)}{J_1(\zeta_r B)H_o^{(1)}(\zeta_r A) - J_o(\zeta_r A)H_1^{(1)}(\zeta_r B)} \tag{22}$$

where n_r is the refractive index of the coating and

$$\zeta_r = (n_r^2 - \alpha^2)^{\frac{1}{2}} \; ; \qquad B = k_1 b$$

In equation (3) for $\tilde{M}_o(\alpha)$, A must be replaced by B, which is now the outer radius of the wire, while A is the radius of the inner conductor.

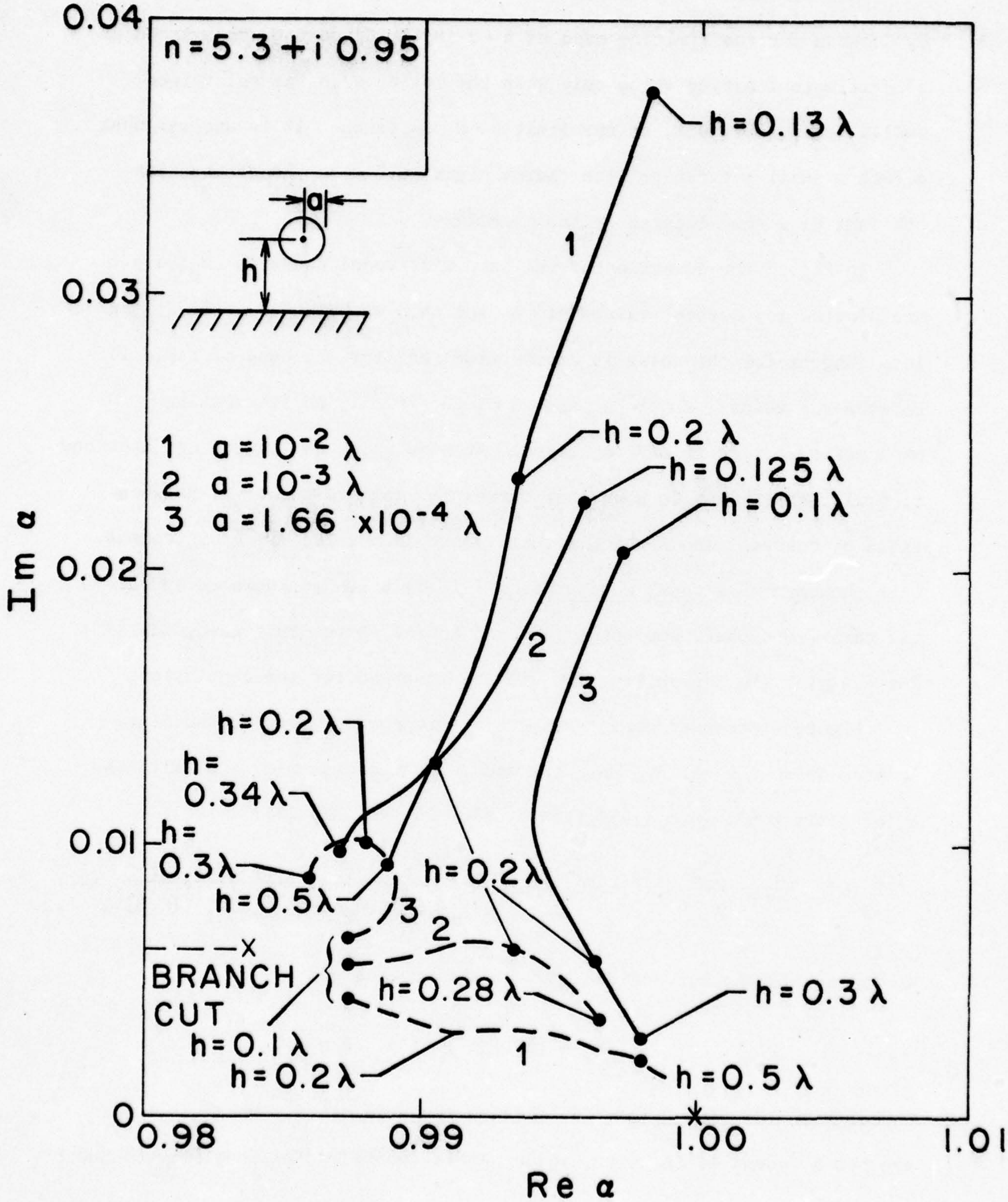


Figure 3

Normalized propagation constant α as a function of wire height h and earth refractive index $n = 5.3 + i0.95$.

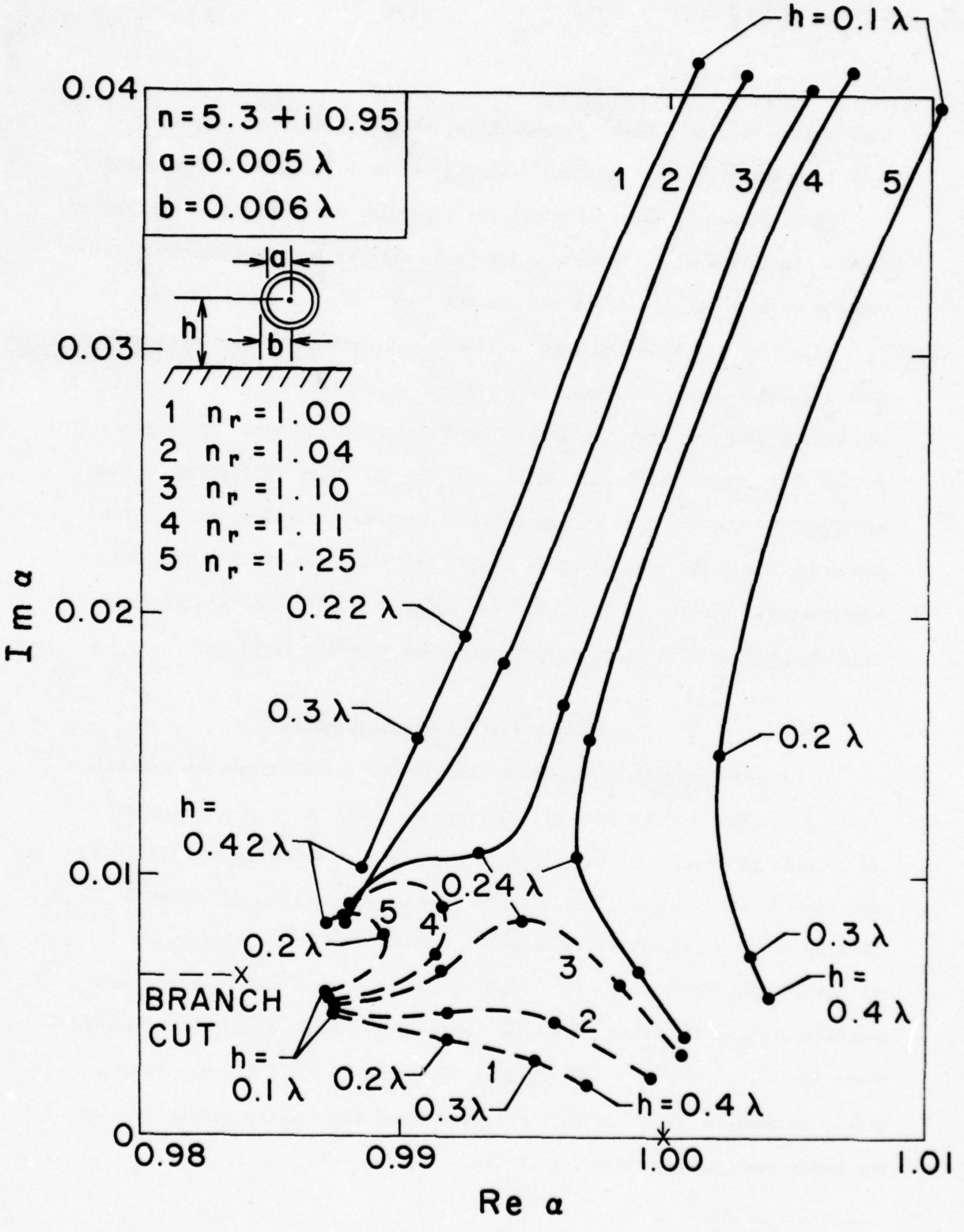


Figure 4

Normalized propagation constant α of a dielectric-coated wire of conductor radius a, coating radius b, coating index n_r, at a height h above earth of refractive index n.

Once again, we find a modal degeneration, this time occurring between n_r = 1.10 and 1.11 (curves 3 and 4 of Fig. 4) at a height of about 0.235λ.

Thus, Kikuchi's conclusion [13,19] regarding the continuous transition from a TEM-like wave to a surface-like wave must be modified somewhat. For the example of Fig. 4, if the coating index n_r is large enough (≥ 1.11), then the TEM-like wave at low height does continuously transform into a surface-like wave (exactly a surface-wave if $h \to \infty$) with increasing h. The second mode (an "earth-attached" or "substrate-attached" mode) remains in the vicinity of the branch point α_B, only slightly influenced by the proximity of the TEM mode at $h \approx 0.24\lambda$. For smaller values of n_r, on the other hand, the mode which is earth-attached at small h transforms continuously into the surface wave for large h, while the "transmission-line" mode transforms into an earth-attached mode for large h.

IV. Field plots for the discrete modes

A computer program to determine the transverse electric and magnetic field distributions for each of the discrete modes has been developed.[4] We present in Figs. 5 - 7 the relative magnitude of the electric field for both discrete modes directly above and below the wire ($y = 0$) as a function of the vertical coordinate x. For the cases $n_r = 1.1, 1.25$, the dotted line corresponds to a mode which has a pronounced surface wave character, while the "earth attached" mode (solid line) decays more slowly above the wire. For $n_r = 1.0$, neither of the modes is distinctly identifiable as surface, "TEM" or earth-attached, and the field distributions of the modes are somewhat less symmetrical.

[4]These are essentially those given in Appendix A.

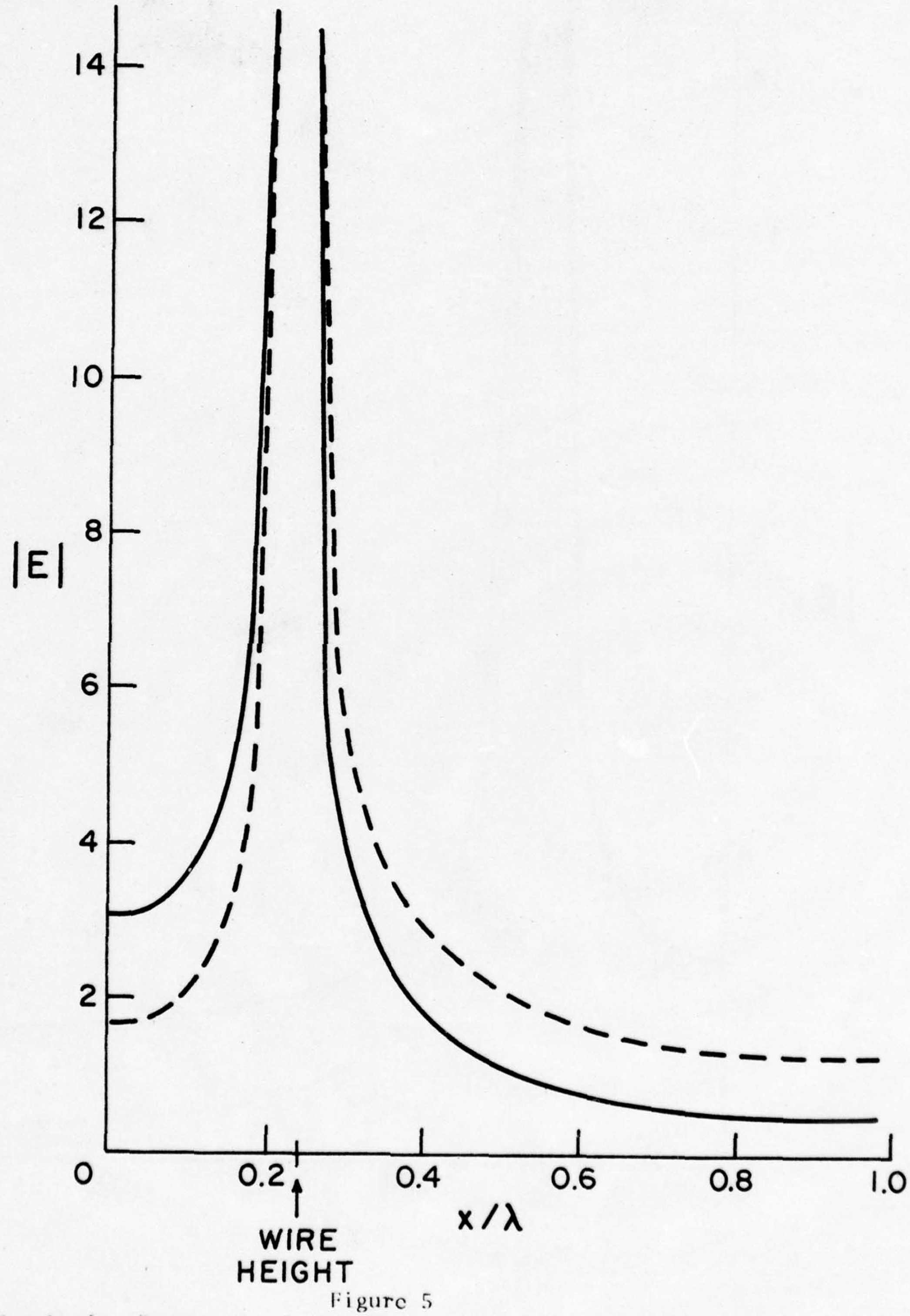


Figure 5

Magnitude of electric field of Goubau line (in relative units) on $y = 0$ axis. $n = 5.3 + i0.45$, $h = 0.24\lambda$, $a = .007\lambda$, $b = .01\lambda$, $n_r = 1.0$, ---- ($\alpha = .99199 + i0.002967$), ——— ($\alpha = .99050 + i0.01545$).

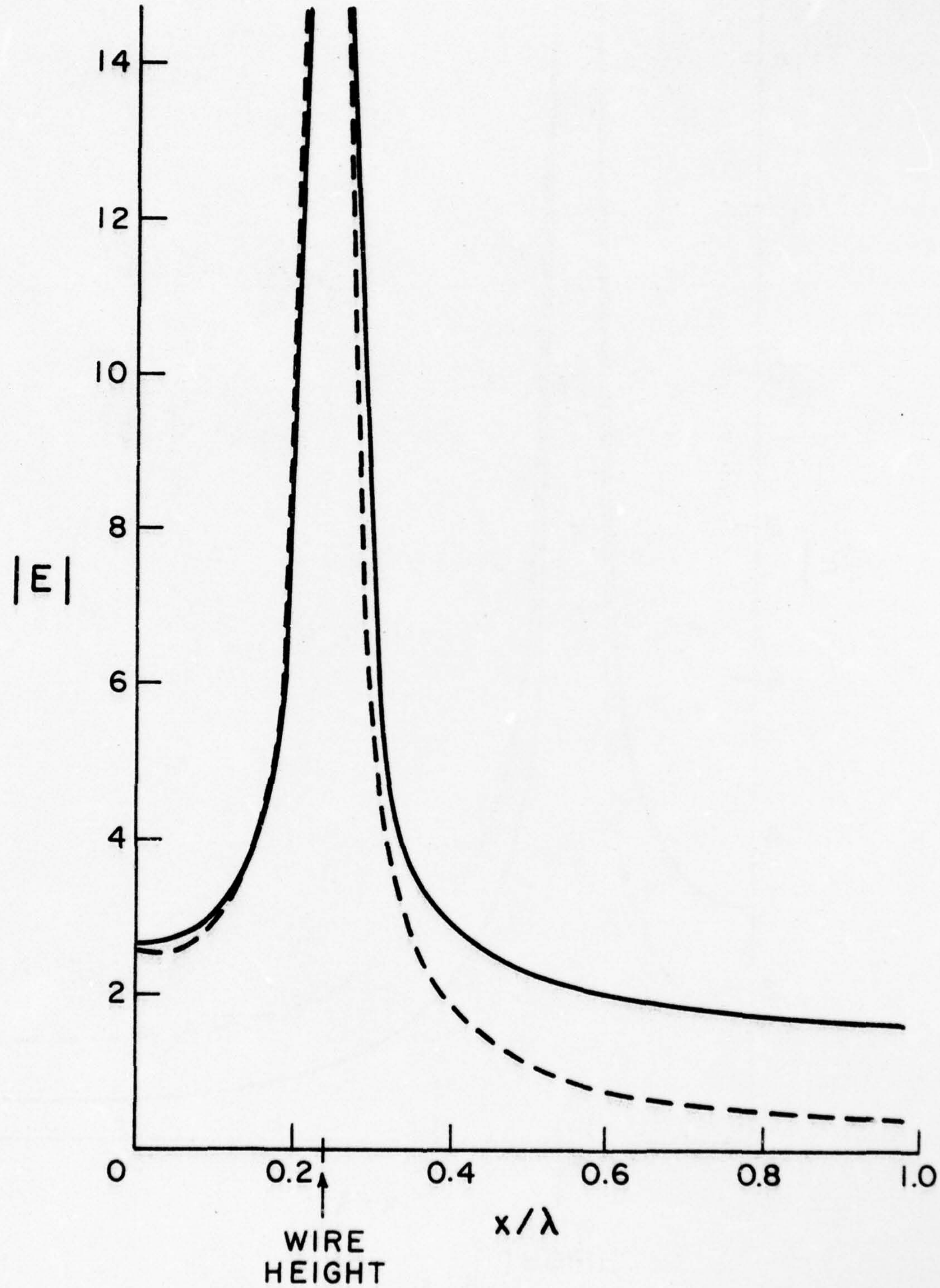


Figure 6
Magnitude of electric field of Goubau line (in relative units) on $y = 0$ axis. $n = 5.3 + i0.45$, $h = 0.24\lambda$, $a = .007\lambda$, $b = .01\lambda$, $a_r = 1.1$, ------ ($\alpha = 1.0019 + i0.01132$), __________ ($\alpha = .98755 + i0.006556$).

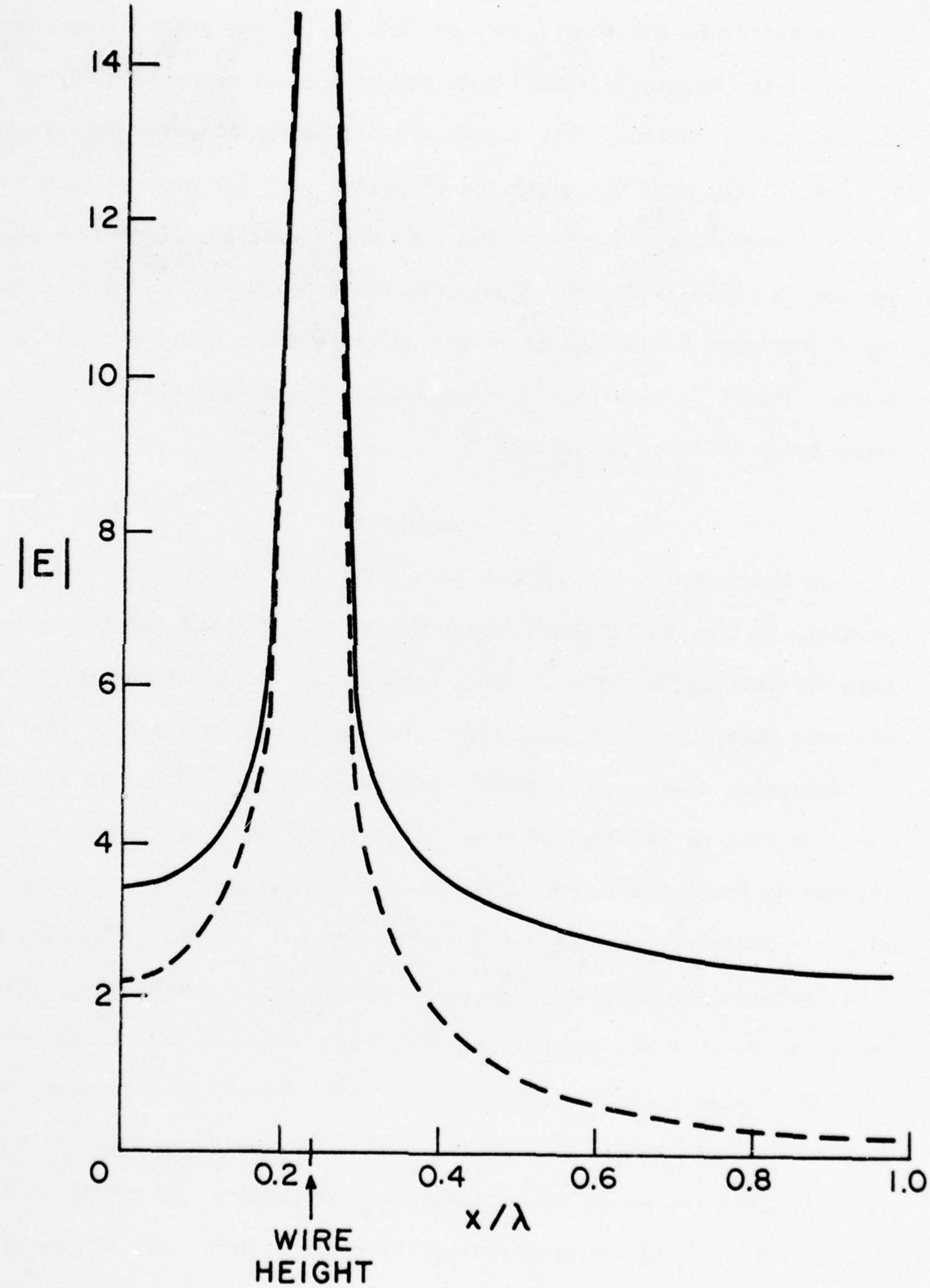


Figure 7
Magnitude of electric field of Goubau line (in relative units) on $y = 0$ axis. $n = 5.3 + i0.45$, $h = 0.24\lambda$, $a = .007\lambda$, $b = .01\lambda$, $n_r = 1.25$, ---- ($\alpha = 1.0123 + 0.01134$), ______ ($\alpha = .9854 + i0.00577$).

In the cross-sectional plots of Figs. 8-13, the greater transverse spread of the "earth-attached" mode can be seen as compared with the transmission-line mode. The fields are all nearly linearly polarized for the latter, with the exception of points very far removed from the wire. These plots, in conjunction with the excitation discussion of section II, allow a design of an efficient experimental excitation scheme for selectively producing one of the discrete modes while minimizing the other. This is a necessary development, if practical utilization of these modes is to be achieved.

V. Conclusion

In this report, a proof has been given that the current on a thin wire parallel to a conducting-half space may be uniquely and completely decomposed into discrete and continuous modal components. In addition, any of the discrete modes may be excited with arbitrarily high efficiency (and to the exclusion of the other modes) by appropriately matching an aperture field to that of the desired mode. This means that even though the continuous or "radiation" type currents must always eventually dominate the discrete modal currents on a lossy structure such as this [28], the discrete contributions can be made to dominate over an arbitrarily long portion of the wire, so that the concept of modal representation will still have important utility in practical applications. Moreover, regardless of which modal contributions (if any) are dominant, the modal representation is always rigorous, and the attendant orthogonality properties are useful in solving problems wherein the wire is not infinite or possesses discontinuities, since such situations can be treated by straightforward generalizations of closed waveguide techniques [26].

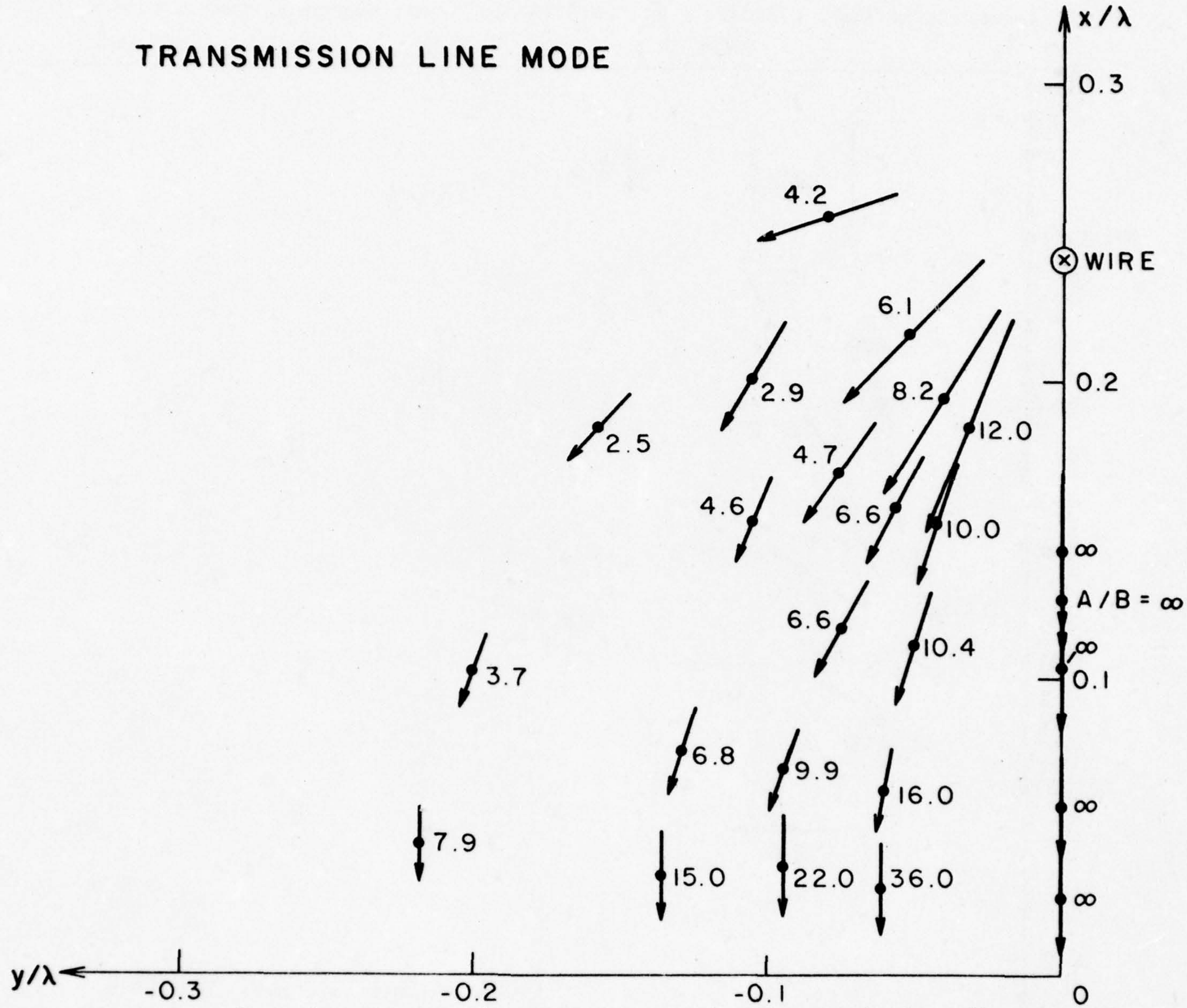


Figure 8. E-field lines for mode on Goubau line. Arrows give field strength along direction of major axis of polarization ellipse. Numbers A/B are ratio of major to minor axis, $n = 5.3 + i0.45$, $h = 0.24\lambda$, $a = .007\lambda$, $b = .01\lambda$, $n_r = 1.0$, $\alpha = .9905 + i0.01545$.

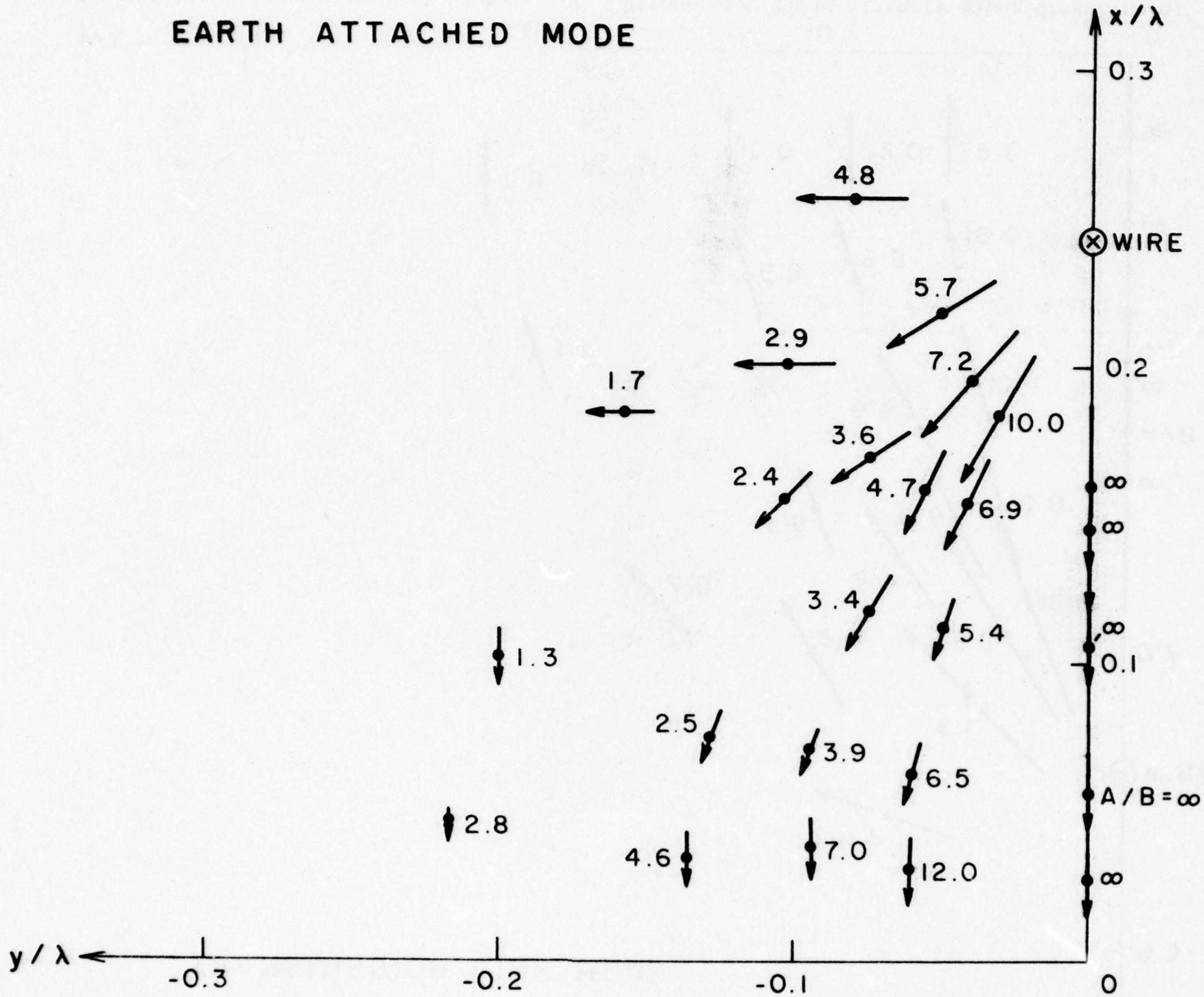


Figure 9. E-field lines for mode on Goubau line. Arrows give field strength along direction of major axis of polarization ellipse. Numbers A/B are ratio of major to minor axis, $n = 5.3 + i0.45$, $h = 0.24\lambda$, $a = .007\lambda$, $b = .01\lambda$, $n_r = 1.0$, $\alpha = .99199 + i0.002967$.

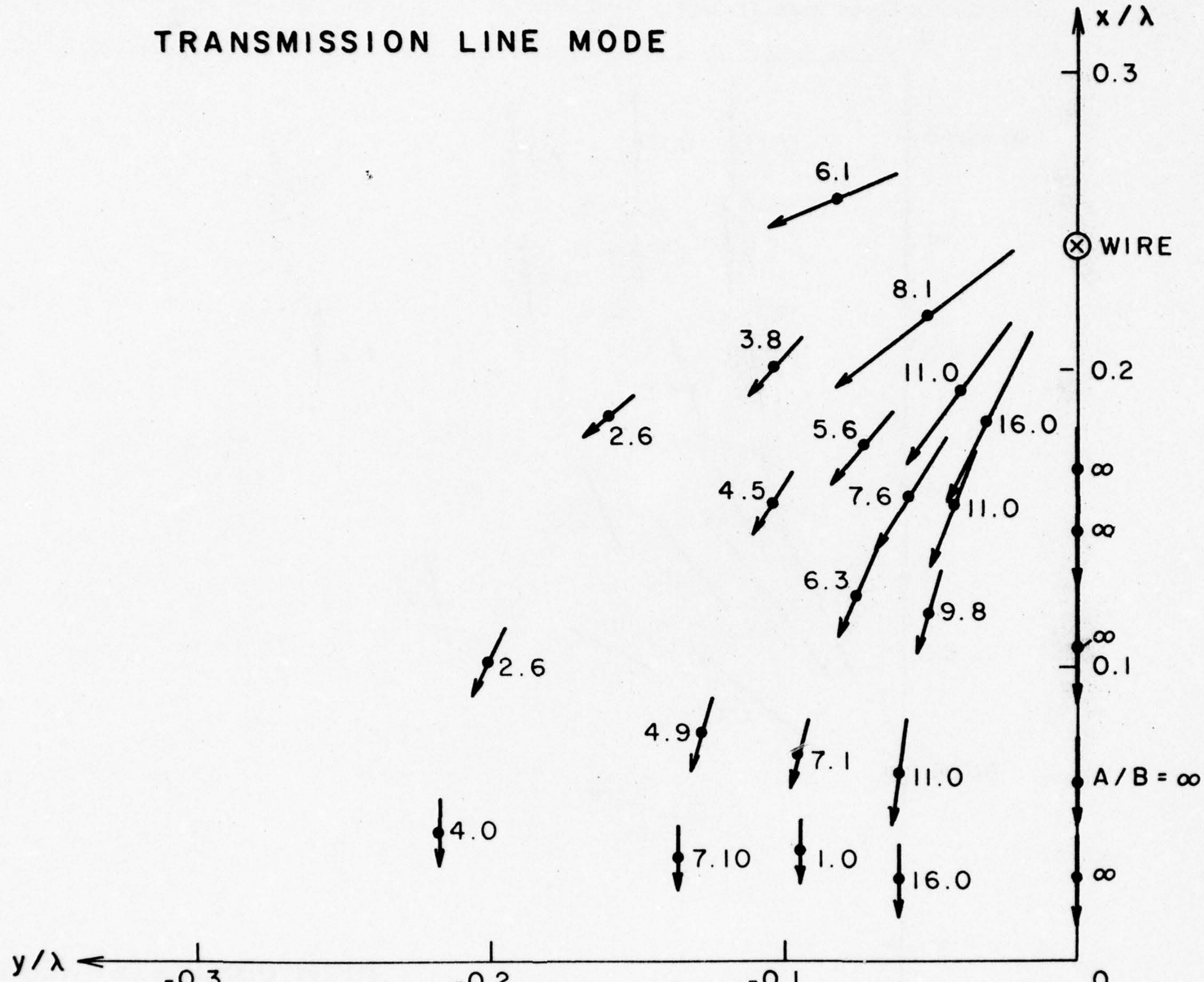


Figure 10. E-field lines for mode on Goubau line. Arrows give field strength along direction of major axis of polarization ellipse. Numbers A/B are ratio of major to minor axis, $n = 5.3 + i0.45$, $h = 0.24\lambda$, $a = .007\lambda$, $b = .01\lambda$, $n_r = 1.1$, $\alpha = 1.0019 + i0.01132$.

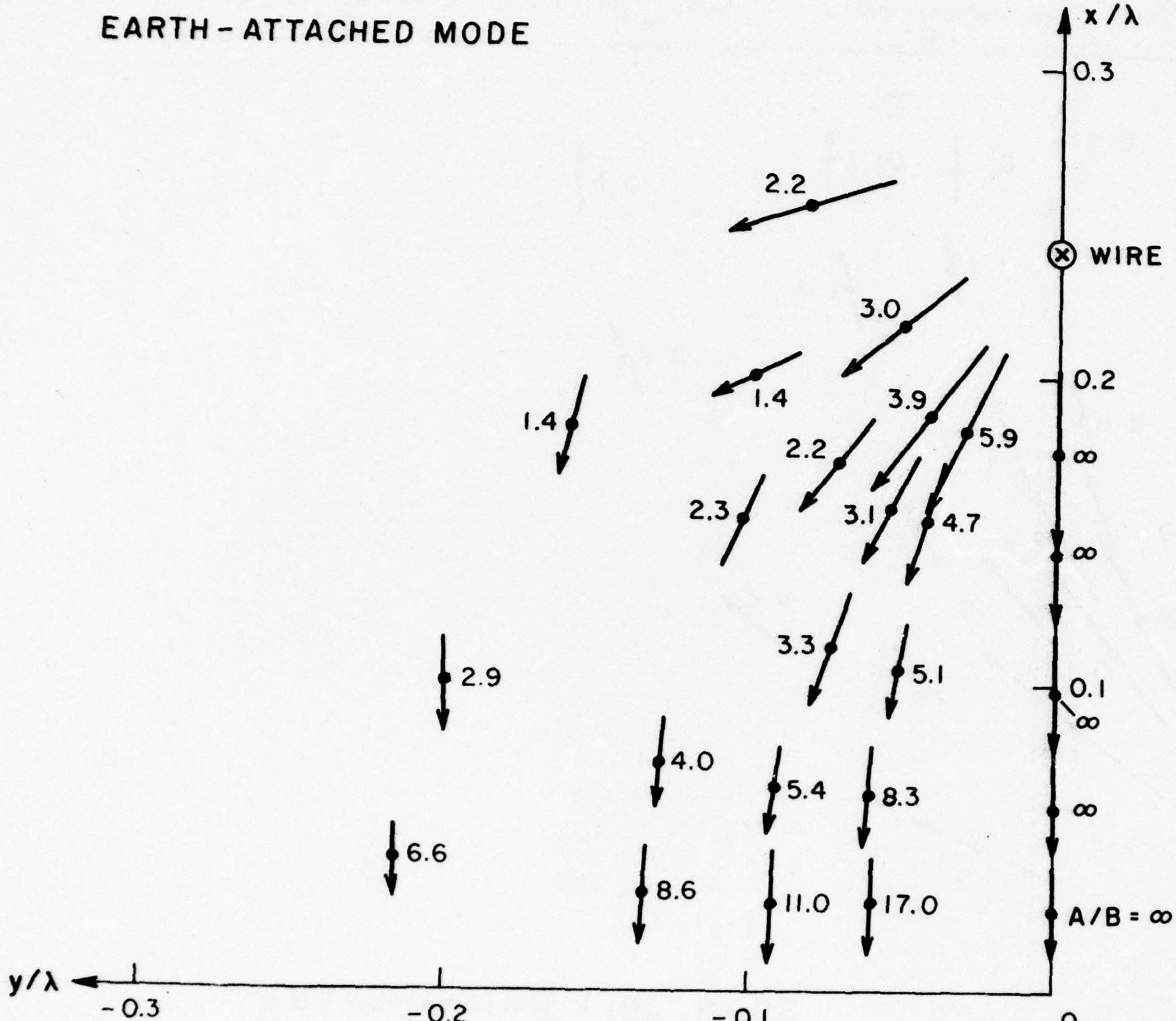


Figure 11. E-field lines for mode on Goubau line. Arrows give field strength along direction of major axis of polarization ellipse. Numbers A/B are ratio of major to minor axis, $n = 5.3 + i0.45$, $h = 0.24\lambda$, $a = .007\lambda$, $b = .01\lambda$, $n_r = 1.1$, $\alpha = .98755 + i0.006556$.

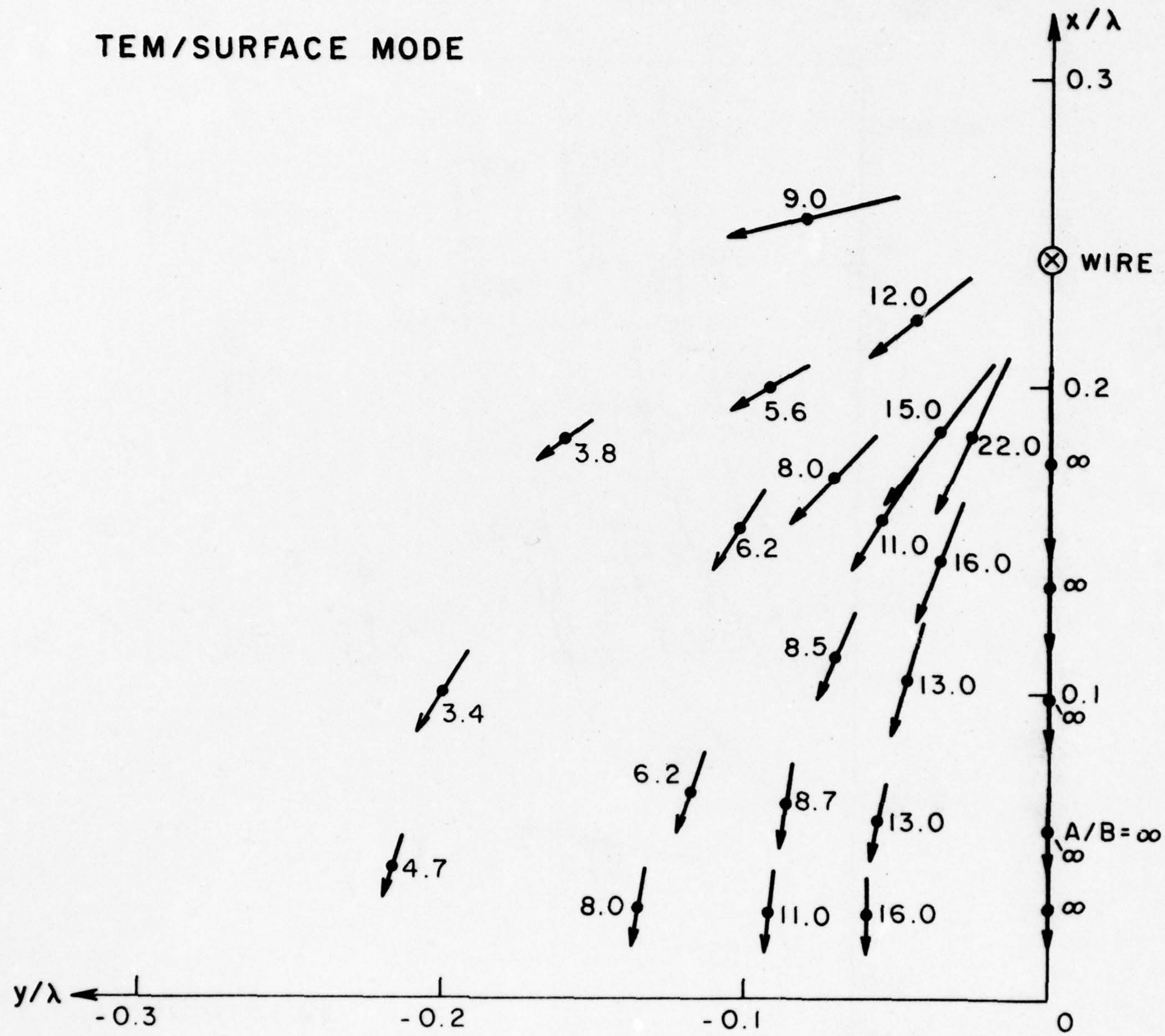


Figure 12. E-field lines for mode on Goubau line. Arrows give field strength along direction of major axis of polarization ellipse. Numbers A/B are ratio of major to minor axis, $n = 5.3 + i0.45$, $h = 0.24\lambda$, $a = .007\lambda$, $b = .01\lambda$, $n_r = 1.25$, $\alpha = 1.0123 + i0.01134$.

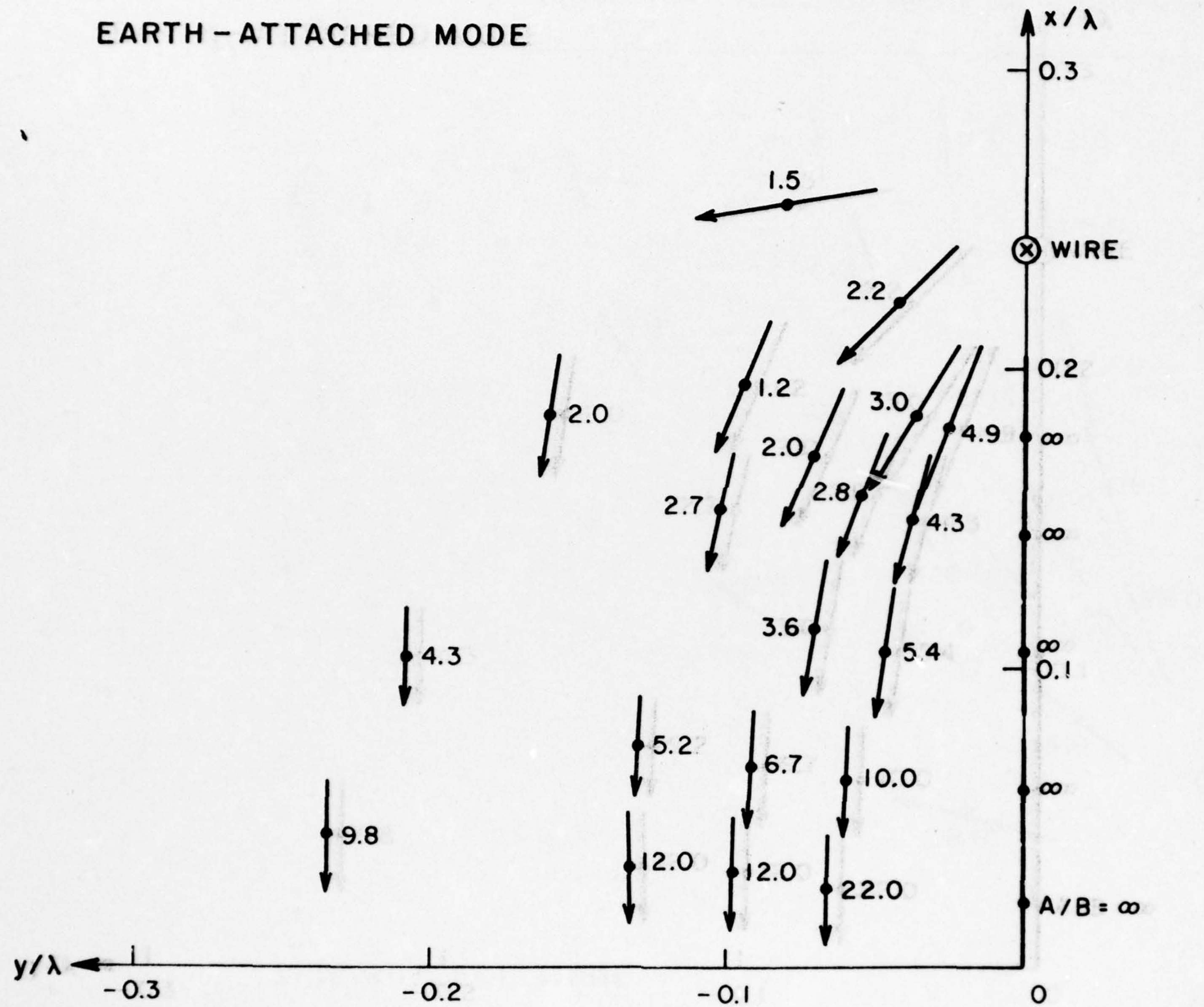


Figure 13. E-field lines for mode on Goubau line. Arrows give field strength along direction of major axis of polarization ellipse. Numbers A/B are ratio of major to minor axis, $n = 5.3 + i0.45$, $h = 0.24$, $a = .007$, $b = .01$, $n_r = 1.25$, $\alpha = 0.98539 + i0.00577$.

A numerical investigation of the behavior of the discrete mode propagation constants, more extensive than those which have appeared previously, shows that the existence of two discrete modes (called "transmission-line" and "earth-attached" in [17]) rather than only one, is not due to any approximation made on the modal equation, but follows even when the exact integral representations of the functions $P(\alpha)$ and (α) appearing therein are used. Moreover, trajectories of the modal propagation constants in the complex α-plane have been given, and the possible existence of modal degeneration has been demonstrated.

It should be emphasized that degeneracy of this kind (algebraic but not geometric, in the language of linear operators [46]) is *not* basically a similar phenomenon to that which occurs, for instance, in closed circular waveguides between TE and TM modes. The latter (which is both algebraic *and* geometric, and could be designated a "semi-simple" mode following Kato [46]) is accidental, a result of the high symmetry of the waveguide, and in this case there remain two orthogonal eigenfunctions (mode functions) corresponding to the same modal eigenvalue. For the nongeometric degeneracy, however, only one modal eigenfunction remains, but the Green's function (resolvent kernel) possesses a double pole at the degenerate eigenvalue.

Perhaps the most commonplace example of a nongeometric degeneracy (though one not usually thought of from this viewpoint) is at cutoff of a mode in a closed waveguide [47]. In this case, the forward-and backward-traveling modes have the same propagation constant (zero) and as a consequence the norm

$$\int_S \bar{E}_n \times \bar{H}_n \cdot \bar{a}_z dS$$

can be shown to vanish, and the excitation equations must be modified in a manner similar to Appendix B.

The question of expanding the fields into modes will be taken up in a future report. In particular, this will require construction of the dyadic (tensor) Green's functions as a generalized product of eigenfunctions, as well as a treatment of the generalized orthogonality and completeness properties of a degenerate discrete mode.

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Appendix A

In this Appendix we present the expressions for the fields $\vec{E}_o^w$ and $\vec{H}_o^w$ of the z-directed electric current source

$$\overline{J}_e = \overline{a}_z \delta(\overline{x}'' - h)\delta(y'')D(z''-z) \tag{A.1}$$

where $D(z'' - z)$ is the generalized function

$$D(z'' - z) = \frac{k_1}{2\pi} \int_{-\infty}^{\infty} J_o(A\zeta) e^{ik_1 \alpha (z''-z)} d\alpha \tag{A.2}$$

The $J_o(A\zeta)$ term in (A.2) reflects an averaging of a dipole source at z over the circumference of the wire (see [14]). The field expressions are found in a straightforward manner (see, for example, [15]) and given in terms of the Fourier transforms as functions of $\overline{x}_t'' = (x'',y'')$:

$$\begin{aligned} \vec{E}_o^w(\overline{x}'';\overline{x}) &= \int_{-\infty}^{\infty} \tilde{\vec{E}}_o^w(\overline{x}_t'';\alpha) e^{+ik_1 \alpha (z''-z)} d\alpha \\ \vec{H}_o^w(\overline{x}'';\overline{x}) &= \int_{-\infty}^{\infty} \tilde{\vec{H}}_o^w(\overline{x}_t'';\alpha) e^{+ik_1 \alpha (z''-z)} d\alpha \end{aligned} \tag{A.3}$$

The transforms are given in terms of two Hertz potentials $\tilde{\pi}_z$ and $\tilde{\pi}_z^*$:

$$\begin{aligned} \tilde{E}_{ox}^w(\overline{x}_t'';\alpha) &= ik_1 \{\alpha \frac{\partial \tilde{\pi}_z}{\partial x''} + \eta_o \frac{\partial \tilde{\pi}_z^*}{\partial y''}\} \\ \tilde{E}_{oy}^w(\overline{x}_t'';\alpha) &= ik_1 \{\alpha \frac{\partial \tilde{\pi}_z}{\partial y''} - \eta_o \frac{\partial \tilde{\pi}_z^*}{\partial x''}\} \\ \tilde{E}_{oz}^w(\overline{x}_t'';\alpha) &= (k^2 - k_1^2 \alpha^2)\tilde{\pi}_z \end{aligned} \tag{A.4}$$

$$\begin{aligned} \tilde{H}_{ox}^w(\overline{x}_t'';\alpha) &= ik_1 \{\alpha \frac{\partial \tilde{\pi}_z^*}{\partial x''} - (\varepsilon_r/\eta_o) \frac{\partial \tilde{\pi}_z}{\partial y''}\} \\ \tilde{H}_{oy}^w(\overline{x}_t'';\alpha) &= ik_1 \{\alpha \frac{\partial \tilde{\pi}_z^*}{\partial y''} + (\varepsilon_r/\eta_o) \frac{\partial \tilde{\pi}_z}{\partial x''}\} \\ \tilde{H}_{oz}^w(\overline{x}_t'';\alpha) &= (k^2 - k_1^2 \alpha^2)\tilde{\pi}_z^* \end{aligned} \tag{A.5}$$

where $k^2 = k_1^2 \varepsilon_r$ and $\varepsilon_r = 1$ or n^2 accordingly as $x>0$ or $x<0$ respectively.

We quote here the results [15]

$$\tilde{\pi}_z = C(\alpha) \int_{-\infty}^{\infty} \frac{\exp(ik_1\lambda y'')}{u_1} \{\exp[-k_1u_1|x'' - h|] + R(\lambda,\alpha)\exp[-k_1u_1(x'' + h)]\}d\lambda \qquad x>0$$

$$= C(\alpha) \int_{-\infty}^{\infty} \frac{\exp(ik_1\lambda y'')}{u_1} T(\lambda,\alpha)\exp[k_1u_2x'' - k_1u_1h]d\lambda \qquad x<0$$

$$\tilde{\pi}_z^* = C(\alpha) \int_{-\infty}^{\infty} \frac{\exp(ik_1\lambda y'')}{u_1} S(\lambda,\alpha)\exp[-k_1u_1(x'' + h)]d\lambda \qquad x>0 \qquad (A.6)$$

$$= C(\alpha) \int_{-\infty}^{\infty} \frac{\exp(ik_1\lambda y'')}{u_1} V(\lambda,\alpha)\exp[k_1u_2x'' - k_1u_1h]d\lambda \qquad x<0$$

where $C(\alpha) = (i\eta_o/8\pi^2)J_o(A\zeta)$

and

$$\left.\begin{aligned}
R(\lambda,\alpha) &= -1 + 2(u_1/\zeta^2)\left[\frac{1}{u_1 + u_2} - \frac{\alpha^2}{u_2 + n^2u_1}\right] \\
T(\lambda,\alpha) &= (\zeta^2/\zeta_n^2)\ [1 + R(\lambda,\alpha)] \\
S(\lambda,\alpha) &= -2(i\lambda\alpha/\eta_o\zeta^2)\left[\frac{1}{u_1 + u_2} - \frac{1}{u_2 + n^2u_1}\right] \\
V(\lambda,\alpha) &= (\zeta^2/\zeta_n^2)\ S(\lambda,\alpha)
\end{aligned}\right\} \qquad (A.7)$$

APPENDIX B

In this appendix we give the derivation for the excitation of the discrete modal currents in the case when the two discrete modes have degenerated $(\alpha_{p1} = \alpha_{p2} = \alpha_p)$. This analysis is along the lines of those in [41,42].

Proceeding from (17), let us find the contribution of the double pole at α_p. Now, the residue of

$$\frac{\tilde{\tilde{H}}^w_o\,(\bar{x}_t''; -\alpha)\,\exp(ik_1\alpha z)}{\tilde{M}(\alpha)}$$

at α_p is given by

$$\frac{\partial}{\partial\alpha}\left\{\frac{(\alpha-\alpha_p)^2\,\tilde{\tilde{H}}^w_o\,(\bar{x}_t''; -\alpha)\,\exp(ik_1\alpha z)}{\tilde{M}(\alpha)}\right\}_{\alpha = \alpha_p} \tag{B.1}$$

$$= \frac{2}{\tilde{M}''(\alpha_p)}\,\frac{\partial}{\partial\alpha}\left[\tilde{\tilde{H}}^w_o\,(x_t''; -\alpha)\exp(ik_1\alpha z)\right]_{\alpha=\alpha_p} - \frac{2\tilde{M}'''(\alpha_p)}{3[\tilde{M}''(\alpha_p)]^2}\left[\tilde{\tilde{H}}^w_o\,(\bar{x}_t''; -\alpha_p)\exp(ik_1\alpha_p z)\right]$$

Thus the pole contribution to (17) is given (for $z > 0$) by:

$$\begin{aligned} I_p(z) = \frac{16\pi i}{\eta_o k_1}\,\frac{\exp(ik_1\alpha_p z)}{M''(\alpha_p)}\Bigg\{ & 2\int_S \bar{a}_z \cdot [\bar{E}^o_t \times \frac{\partial}{\partial\alpha}\,\tilde{\tilde{H}}^w_o\,(\bar{x}_t''; -\alpha)_{\alpha=\alpha_p}]\,dS'' \\ & + 2\left(ik_1 z - \frac{1}{3}\frac{M'''(\alpha_p)}{M''(\alpha_p)}\right)\int_S \bar{a}_z \cdot [\bar{E}^o_t \times \tilde{\tilde{H}}^w_o\,(\bar{x}_t''; -\alpha_p)]\,dS''\Bigg\} \end{aligned} \tag{B.2}$$

In addition to the contribution from the ordinary discrete modal field $\tilde{\tilde{H}}^w_o\,(\bar{x}_t''; -\alpha_p)$ at the aperture, there is a contribution from the field

$$\frac{\partial}{\partial\alpha} \tilde{H}_o^w (x_t''; -\alpha)_{\alpha=\alpha_p}$$

which does not have the behavior of the ordinary mode (the fields will be somewhat more spread out). Following the nomenclature of the mathematical theory of linear operators, this may be termed an "adjoined mode" [41]. It should also be noted that the excitation of the ordinary modal current possesses a factor growing linearly with z. This, of itself, does not imply any violation of conservation of energy, since a mode at degeneracy is self-orthogonal [41, 42] and energy is readily exchanged between it and the adjoined mode.

APPENDIX C

In this Appendix we show that when the aperture field is replaced by the modal field for one of the discrete modes, then all the terms of (17) vanish with the exception of the corresponding discrete modal current. These arguments are based on orthogonality proofs by R.G. Olsen [48] which are similar to those given for related problems in [49-51].

Suppose, for definiteness, that $\bar{E}_t^o = \bar{E}_{t1}$, corresponding to the propagation constant α_{p1}. Now the field $\bar{H}_{t2}^- = \tilde{\bar{H}}_o^w\ (\bar{x}_t;\ -\alpha_{p2})$ is a transpose modal field (propagating in the -z direction) whose fields are related to the forward mode by (see A.4 and A.5 in Appendix A)

$$\bar{E}_{t2}^- = -\bar{E}_{t2}^+\ ;\quad E_{z2}^- = +E_{z2}^+\ ;\quad \bar{H}_{t2}^- = +\bar{H}_{t2}^+\ ;\quad H_{z2}^- = -H_{z2}^+ \tag{C.1}$$

Both discrete modal fields satisfy Maxwell's equations with no sources, as well as the boundary conditions (in the uniform-current approximation) at the wire.

Using the Lorentz reciprocity relation

$$\nabla \cdot (\bar{E}_1^+ \times \bar{H}_2^- - \bar{E}_2^- \times \bar{H}_1^+) = 0$$

where $\nabla = \nabla_t + ik_1(\alpha_{p1} - \alpha_{p2})\bar{a}_z$, we integrate over a cross-section S of large radius, and apply the divergence theorem to the portion involving ∇_t. Because of the exponential decay of the integrand in the transverse direction, the resulting line integral

$$\oint_C \bar{a}_n \cdot (\bar{E}_1^+ \times \bar{H}_2^- - \bar{E}_2^- \times \bar{H}_1^+) d\ell$$

tends to zero as the boundary C of S recedes to infinity. Thus in the limit as S becomes the entire (infinite) cross-section, we have

$$\int_S \bar{a}_z \cdot (\bar{E}_1^+ \times \bar{H}_2^- - \bar{E}_2^- \times \bar{H}_1^+) dS = 0 \qquad (\alpha_{p1} \neq \alpha_{p2}) \qquad \text{(C.2)}$$

In a similar manner, one shows that

$$\int_S \bar{a}_z \cdot (\bar{E}_1^+ \times \bar{H}_2^- - \bar{E}_2^- \times \bar{H}_1^+) dS = 0 \qquad (\alpha_{p1} \neq -\alpha_{p2}) \qquad \text{(C.3)}$$

combined with (A.1) and (A.2), (A.3) shows that the integral I_{p2} in (18) vanishes.

If it is noted that all that was used in (A.1) - (A.3) was the fact that the fields satisfied Maxwell's equations and the boundary conditions, and that the integrands decayed exponentially at infinity, we can see easily that the integrals I_{B1} and I_{B2} must vanish in precisely the same way. Thus, if the aperture field could precisely match that of one of the discrete modes, the only contribution to the current on the wire would be that of the same discrete mode.